Prime Test Without Division

Prime Testing

A natural number n is *prime* if it has no divisors other than 1 and n itself. A common test for primeness is to try dividing $\frac{n}{d}$ for natural numbers d greater than 1 and less than or equal to $\sqrt{n}$. It is possible, however, to test whether n is prime without doing any divisions. In fact, multiplications, square roots, and even subtractions may be avoided.

Table of Squares

The algorithm depends on a table of squares $s_i = i^2$. The squares may be generated as sums of odd numbers to avoid multiplication. For example, $4^2 = 1 + 3 + 5 + 7$.

The Algorithm

Let n be an odd number to test for primeness, and follow Algorithm 1.

Algorithm 1

```
    Input: odd number n to test for primeness
    Let  i = 0
    Let  j = 1
    Loop:

        If  s_i + n > s_j then increment  j
        Else If  s_i + n < s_j then increment  i
        Else terminate

    On terminate:

        If  i + 1 = j then  n is prime
```